Let $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$ and $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ be bases for U and V (respectively). Then, the set $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3, \boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ is linearly dependent, since ⟨acronymref|theorem|G⟩ says we cannot have 6 linearly independent vectors in a vector space of dimension 5. So we can assert that there is a non-trivial relation of linear dependence,

$$a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + a_3\boldsymbol{u}_3 + b_1\boldsymbol{v}_1 + b_2\boldsymbol{v}_2 + b_3\boldsymbol{v}_3 = \boldsymbol{0}$$

where a_1, a_2, a_3 and b_1, b_2, b_3 are not all zero. We can rearrange this equation as

$$a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + a_3\boldsymbol{u}_3 = -b_1\boldsymbol{v}_1 - b_2\boldsymbol{v}_2 - b_3\boldsymbol{v}_3$$

This is an equality of two vectors, so we can give this common vector a name, say $\boldsymbol{w}$,

$$\boldsymbol{w} = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + a_3\boldsymbol{u}_3 = -b_1\boldsymbol{v}_1 - b_2\boldsymbol{v}_2 - b_3\boldsymbol{v}_3$$

This is the desired non-zero vector, as we will now show. First, since $\boldsymbol{w} = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + a_3\boldsymbol{u}_3$, we can see that $\boldsymbol{w} \in U$. Similarly, $\boldsymbol{w} = -b_1\boldsymbol{v}_1 - b_2\boldsymbol{v}_2 - b_3\boldsymbol{v}_3$, so $\boldsymbol{w} \in V$. This establishes that $\boldsymbol{w} \in U \cap V$ (⟨acronymref|definition|SI⟩).Is $\boldsymbol{w} \neq \boldsymbol{0}$? Suppose not, in other words, suppose $\boldsymbol{w} = \boldsymbol{0}$. Then

$$\boldsymbol{0} = \boldsymbol{w} = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + a_3\boldsymbol{u}_3$$

Because $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$ is a basis for U, it is a linearly independent set and the relation of linear dependence above means we must conclude that $a_1 = a_2 = a_3 = 0$. By a similar process, we would conclude that $b_1 = b_2 = b_3 = 0$. But this is a contradiction since a_1, a_2, a_3, b_1, b_2, b_3 were chosen so that some were nonzero. So $\boldsymbol{w} \neq \boldsymbol{0}$. How does this generalize? All we really needed was the original relation of linear dependence that resulted because we had "too many" vectors in W. A more general statement would be: Suppose that $\boldsymbol{w}$ is a vector space with dimension n, U is a subspace of dimension p and V is a subspace of dimension q. If $p + q > n$, then $U \cap V$ contains a non-zero vector.

Sean $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$ y $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ bases para $U y V$ (respectivamente). Luego, el conjunto $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3, \boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ es linealmente independiente, ya que ⟨acronymref|theorem|G⟩ dice que no podemos tener 6 vectores linealmente independientes en un espacio vectorial de dimension 5. Por lo tanto, podemos afirmar que existe una relacion no trivial de dependencia lineal,

$$a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + a_3\boldsymbol{u}_3 + b_1\boldsymbol{v}_1 + b_2\boldsymbol{v}_2 + b_3\boldsymbol{v}_3 = \boldsymbol{0}$$

donde a_1, a_2, a_3 y b_1, b_2, b_3 no son todos ceros. Podemos cambiar esta ecuación como

$$a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + a_3\boldsymbol{u}_3 = -b_1\boldsymbol{v}_1 - b_2\boldsymbol{v}_2 - b_3\boldsymbol{v}_3$$

Esto es una igualdad de dos vectores, por lo cual podemos darle al vector común un nómbre, llamado $\boldsymbol{w}$

$$\boldsymbol{w} = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + a_3\boldsymbol{u}_3 = -b_1\boldsymbol{v}_1 - b_2\boldsymbol{v}_2 - b_3\boldsymbol{v}_3$$

Este es el vector deseado diferente de cero, como se muestran ahora. Primero, desde $\boldsymbol{w} = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + a_3\boldsymbol{u}_3$, podemos ver que $\boldsymbol{w} \in U$. Igualmente $\boldsymbol{w} = -b_1\boldsymbol{v}_1 - b_2\boldsymbol{v}_2 - b_3\boldsymbol{v}_3$, de modo que $\boldsymbol{w} \in V$. Esto establece que $\boldsymbol{w} \in U \cap V$ (⟨acronymref|definition|SI⟩). ¿Es $\boldsymbol{w} \neq \boldsymbol{0}$? Suponga que no, en otras palabras, suponga $\boldsymbol{w} = \boldsymbol{0}$. Luego

$$\boldsymbol{0} = \boldsymbol{w} = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + a_3\boldsymbol{u}_3$$

Dado que $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$ es una base de U, es un conjunto linealmente independiente y la relación de dependencia lineal anterior significa que debemos concluir que $b_! = b_2 = b_3 = \mathbf{0}$. Pero es una contradicción desde a_1, a_2, a_3, b_1, b_2, b_3 fueron elegidos tal que fueran diferentes de cero. Asi $\boldsymbol{w} \neq \mathbf{0}$. ¿Como se puede generalizar? Todo lo que realmente se necesitaba era la original relación de dependencia lineal que dio lugar, ya que había "demasiados" vectores en W. Una declaración más general sería: Suponga que $\boldsymbol{w}$ es un espacio vectorial de dimension n, U es un subespacio de dimension p y V es un subespacio de dimension q. Si $p + q > n$, luego $U \cap V$ contiene un vector diferente de cero.

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